MATH 54 – MIDTERM 1 STUDY GUIDE

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Note: Midterm 1 is on **Friday, June 29th** in 4 Evans from 12:05 pm to 1 pm (although I will try to give you more time!) It covers sections $1.1 - 1.5, 2.1 - 2.3, 3.1 - 3.2$ of the Linear Algebra book, although there will be **NO** questions on linear combinations, span, and linear (in)dependence (except for the IMT in section 2.3).

Note: 1.3.4 means 'Problem 4 in section 1.3'

CHAPTER 1: LINEAR EQUATIONS IN LINEAR ALGEBRA

- Solve a system of equations, or determine if there are no solutions. (1.1.11, 1.1.13, 1.1.15, 1.2.7, 1.2.11, 1.2.13)
- Solve the equation $A\mathbf{x} = \mathbf{b}$ for a given $\mathbf{b}$ (1.4.11, 1.5.1, 1.5.3, 1.5.9. 1.5.11, 1.5.13)
- Also know how to write your answer in parametric vector form. For example, know how to write your solution in the form:

$$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + s \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + t \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

- Know the 'useful fact' about inconsistent systems (i.e. a system has no solutions if and only if there is a row in the augmented matrix of the form $\begin{bmatrix} 0 & 0 & 0 & 0 & b \end{bmatrix}$, where $b \neq 0$ (1.3.23, 1.3.24)
- Know the fact that the general solution to $A\mathbf{x} = \mathbf{b}$ is the sum of a particular solution to $A\mathbf{x} = \mathbf{b}$ plus the general solution to $A\mathbf{x} = \mathbf{0}$.

CHAPTER 2: MATRIX ALGEBRA

- Given matrices A and B, calculate AB, BA, A^2, A^T, $A + B$, $-3A$ etc. (2.1.1, 2.1.3)
- Find the inverse of a 2×2 matrix using the formula on page 121 (2.2.1, 2.2.3)
- Prove a couple of cute facts about invertible matrices (see 2.2.13 or 2.2.15 nothing more difficult than that)
- Find the inverse of any matrix (or show it is not invertible) using row-reduction (2.2.31, 2.2.32)
- Know a couple of facts about inverses (such as $(A^{-1})^{-1} = A, (AB)^{-1} = B^{-1}A^{-1}$)
- **Know** conditions $(a), (b), (c), (d), (e), (g), (h)$ of the invertible matrix theorem! (page 131). That's the only time I will ask you about linear independence (see note on the next page)

Date: Friday, June 29th, 2012.

Note: You **don't** need to learn the IMT by heart, just remember that invertible matrices are awesome! For example, I could ask you: Is the following matrix invertible?

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

And you would tell me 'No because its columns are linearly dependent, hence by IMT the matrix is not invertible'

Or I could ask you: Is a 3×3 matrix with 2 pivots invertible (No).

- Use the IMT to figure out if a matrix is invertible or not, or other facts about invertible matrices (2.3.3, 2.3.5, 2.3.8, 2.3.15, 2.3.18, 2.3.22, 2.3.24).

CHAPTER 3: DETERMINANTS

- Calculate the determinant of a 2×2 matrix using the formula: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
- Calculate the determinant of an $n \times n$ matrix using the algorithm shown to you on Monday (3.1.1, 3.1.3, 3.1.9, 3.1.13)
- Calculate the determinant of an $n \times n$ matrix using row-reduction (3.2.5, 3.2.7)
- Use determinants to figure out if a matrix is invertible or not (3.2.23)
- Know how to use the formula $det(AB) = det(A)det(B)$ and $det(A^{-1}) = \frac{1}{det(A)}$ (3.2.31, 3.2.34, 3.2.36)

TRUE/FALSE EXTRAVAGANZA

Check out the following set of T/F questions (solutions are in the HW hints, but beware, there might be mistakes, e-mail me whenever something seems to be wrong): 1.1.23(a)(b), 1.1.24, 1.2.21, 1.3.23(d), 1.3.24(a)(b), 1.4.23(c)(f), 1.4.24(c)(e), 1.5.23(a)(c)(e), 1.5.24(a)(c)(e), 2.3.11, 2.3.12(a)(b)(c), 3.2.27, 3.2.28

CONCEPTS

Here are a couple of concepts we learned so far. You **don't** have to memorize the definitions, just have a rough idea of what those things are

- Pivots
- (In)consistent systems
- (Reduced) row-echelon form
- Elementary row operations
- Vector
- Free variable
- Homogeneous equation
- A^T
- A^{-1}, Invertible matrix, Invertible Matrix Theorem
- Determinants